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ON THE APPLICATION OF PARTICLE SWARM OPTIMIZATION IN MINIMUM TIME LAUNCH VEHICLE TRAJECTORY OPTIMIZATION PROBLEM

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ABSTRACT

The application of Particle Swarm Optimization (PSO), in solving minimum time trajectory optimization problem, studied in this paper. The ascent phase of a launch vehicle trajectory is considered, due to its highly nonlinear nature. It is one of the challenging problems in optimization scenario. Achieving the target in minimum time and error, satisfying all the constraints is the major objective. A highly nonlinear complex system, launch vehicle, is considered with the control parameter as its angle of attack. It is proved that, PSO approach outperforms other such type of optimization, procedures in trajectory optimization scenario, by providing better accuracy. The numerical results are evaluated, analyzed and presented in a MATLAB simulation environment.

KEYWORDS: Minimum Time Problem, Trajectory Optimization & Particle Swarm Optimization (PSO)

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INTRODUCTION

The launch vehicle trajectory is generally optimized, to meet the target by minimum fuel, minimum control or minimum time. The objective of optimization considered in this research, is to optimize the time taken by the launch vehicle to reach the right injection point, satisfying all the set constraints. This process provides an economic consideration, as well as performance enhancement. There is a vast amount of literature available, in the area of launch vehicle trajectory optimization.

Lukkana et al. [1], gives a detailed discussion on the minimum control approach, for the launch vehicle and in Mauro et. al. [2], a multi-stage launch vehicle trajectory is discussed. Many numerical procedures exist to solve the optimization problem. Gradient restoration algorithm [3] and shooting method [4], are the indirect methods, whereas direct collocation [5] and differential inclusion [6], are the few examples of direct methods. One of the standard procedures, to solve the nonlinear equations is a gradient or steepest-decent method. In Sirisha et al. [8] this method is applied, to a hyper-sonic launch vehicle. In Vathsal, et al. [9] min-max technique is used for a satellite launch system, to obtain an optimum pitch angle, so as to maximize the apogee velocity at a constraint altitude and perigee point values.

The evolutionary bio inspired techniques are very effective in solving such problems, due to their easy to handle nature. The implementation of such algorithms is comparatively less complicated [22]. The PSO algorithm proposed by Kennedy and Eberhart, in 1995 proved to be one of the best candidates, for solving stochastic problems. The algorithm works on the basis of best behavior, among various entities in a social system. The previous paper published by the authors, gives an insight on solving the same optimization problem, using

non-evolutionary method such as steepest descent. In this paper, a comparison is also performed and it is clearly identified that, PSO algorithm outperforms steepest descent method, in solving minimum time problems that is considered in this paper.

The rest of the paper is organized as follows. An overview of the optimization problem formulation is discussed in section II. PSO and its application in the proposed system are given in section III. In section IV, simulation results and analysis are discussed, followed by conclusions inferred in section V, along with some views and future scope.

PROBLEM FORMULATION

This section gives details about the mathematical modelling of the launch vehicle equation of motion. [21].

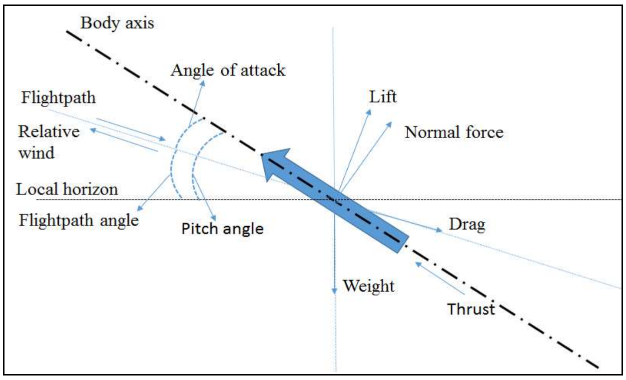

Figure.1: Force Components for an Air/Space Vehicle [21]

Dynamics of the System

As explained earlier, a highly nonlinear system, a launch vehicle is considered in this research. With reference to the available literature, a reference systems Advanced Launch vehicle System model (ALS), developed by NASA [19] is taken for the analysis, for the proposed methodology. Point mass equation of motion with spherical earth is given by,

$$\dot{r} = V \sin \gamma \tag{1}$$

$$\dot{V} = \frac{1}{mV}(T \cos \alpha - D - mg \sin \gamma) \tag{2}$$

$$\dot{\gamma} = \frac{1}{mV}(T \cos \alpha + L) + (\frac{V}{r} - \frac{g}{V}) \cos \gamma \tag{3}$$

Where, m, T, D and L refers to the mass of the launch vehicle, thrust, drag and lift, respectively, and g is the acceleration due to gravity.

The parameters r and v, distance from centre of the earth and velocity, respectively and the flight path angle are the state variables, along with the angle of attack as the control parameter.

In this paper, minimum time optimization is considered. For that, the time dependent equations have to be converted into flight path angle dependent, so as to make flight path, a dependent variable.

$$\dot{r} = F(\gamma)\,; \dot{v} = G(\gamma)\,;\ \gamma = I(\gamma) \tag{4}$$

Physical Constraints

Constraints refer to functions that define the allowable limits of the respective variables. The constraints considered in this research are stated below.

- At the injection point, there must be a minimum error in state variables considered. Also, the accuracy of the achieved velocity, with respect to the desired one should be very high.
- Since, this is a minimum time optimization approach, the trajectory generated must be in such a way that, the vehicle reaches the injection point, in minimum possible time.

Performance Measure

The performance measure is selected, for evaluating the system performance and is denoted as J throughout the paper, which is also called as cost function. Here, the problem is a combination of minimum-time and minimum terminal error problems; the performance measure is given by [12]:

$$J = \sum S_X \left\| x_f - x_{t_f} \right\|_2^2 + \int_{t_0}^{t_f} dt \tag{5}$$

Where, s_r s_v and s_γ are the weighing factors. They are properly selected for each constraint. To reformulate the performance measure, γ is chosen as an independent variable. Therefore, the cost function can be substituted as,

$$J = \sum S_X \left\| x_f - x_{\gamma_f} \right\|_2^2 + \int_{\gamma_0}^{\gamma_f} \frac{dt}{d\gamma} d\gamma \tag{6}$$

PARTICLE SWARM OPTIMIZATION ALGORITHM FOR SOLVING MINIMUM TIME PROBLEM

PSO is proposed by Kennedy and Eberhart in 1995, based on the behavior of social systems, such as fish schooling, bird flocking etc. Since, it is taking the best of all the entities in the system, it is better than other stochastic methods. It is one of the widely accepted and applied evolutionary optimization methodologies.

Consider that, i represents the total number of particles and D is the degree of freedom. i^{th} particle in D^{th}-dimension is denoted as, $Xi = (x_{i1}, x_{i2}, ... x_{iD})^T$. The velocity of each particles are represented by, $Vi = (v_{i1}, v_{i2}, ... v_{iD})$ T. Each particle has individual personal best positions and is denoted as, $Pi = (p_{i1}, p_{i2}, ..., p_{iD})^T$. The best value obtained in each iteration is called the global best solution ('g' in the equations).

Equation (7) is used to update the velocity and position of the particles in global, version PSO, in which the superscripts denote the iteration number [16].

$$v_i^{t+1}(d) = \omega v_i^{t+1}(d) + c_1 rand(p_i^t(d) - x_i^t(d) + c_2 rand(p_g^t(d) - x_i^t(d)) \; x_i^{t+1}(d) = x_i^t(d) + v_i^{t+1}(d) \tag{7}$$

Where, $x_i^t(d)$ and $v_i^t(d)$ denotes current position and velocity of the d^{th} dimension of the i^{th} particle, and r denotes the uniformly distributed random number $U \in [0,1]$. c_1 and c_2 are positive acceleration constants. ω is the inertia weight.

Due to the parameter dependent nature and the constant inertia weight, the classical PSO suffers from local

optima locking, which may even result in particles going out of the boundary. To alleviate these shortcomings, Eberhart *et.al* [17] proposed a new version of PSO called Adaptive PSO (APSO). In this version, the inertia weight parameter is initially given a higher value and it goes on diminishing, with each iteration. In APSO, the adaptive inertial weight is updated, based on the following equation.

$$\omega^{t+1} = \omega_{\max} - \left(\frac{\omega_{\max} - \omega_{\min}}{\max\ iter}\right) iteration \tag{8}$$

Generally $\omega_{\max}$ and **Error! Bookmark not defined.** values are usually fixed as 0.9 and 0.4.

SIMULATION RESULTS AND DISCUSSIONS

The minimum time problem is solved, by minimizing terminal error of a constant thrust engine launch vehicle trajectory. The control variable considered is the angle of attack. It is always kept under the allowable limits, so as not to increase beyond the critical limit. Table 1 gives target parameters considered [19].

Table 1: Desired Performance Parameters

Parameters	Initial Conditions	Final Conditions
r	6377353	6438553
v	65	2630
γ	89.5	0
m	1523400	152052
t	0	150

The simulation is done in Matlab environment and corresponding results are given below. The best solutions obtained after doing 50 iterations are presented in figures 2 to 5. The parameters for the PSO are taken from Mengqi et al. [20]. Figure 2, represents the variation of control vector, with respect to time. Figure 3, shows the velocity attained by the launch vehicle, during each time instant of its flight. From the graph, it is achieving the target velocity in 147.6 seconds. Figure 4 and 5, represents the variation of flight path angle and the radial distance from centre of earth, respectively. From the results it is clear that, the injection criteria are met satisfactorily.

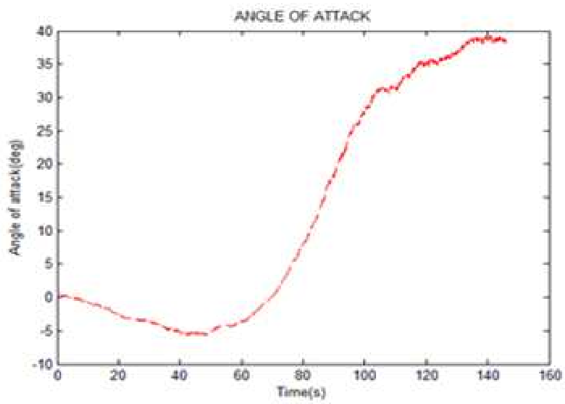

Figure 2: Variation of Control Variable

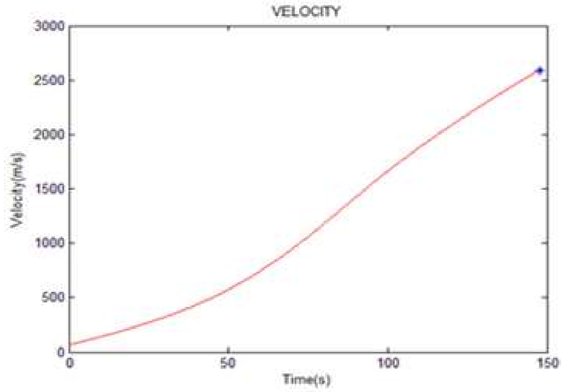

Figure 3: Velocity Profile of the Launch Vehicle

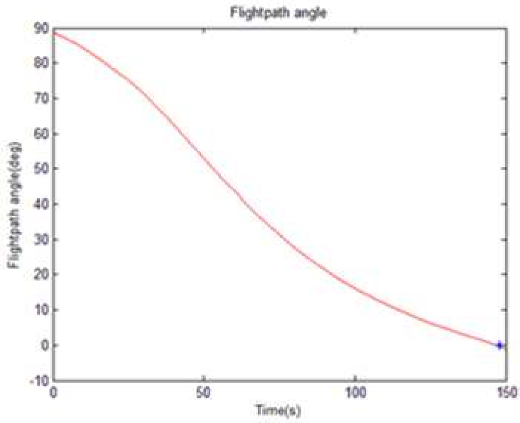

Figure 4: Variation of Flight Path Angle

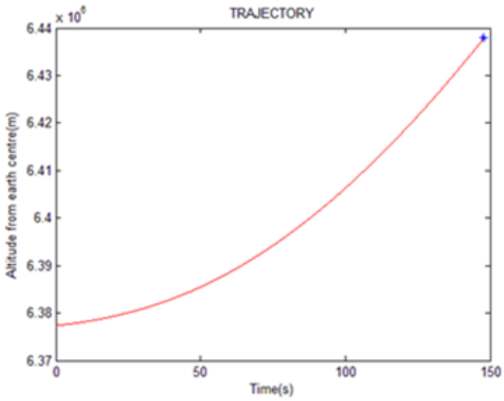

Figure 5: Radial Distance Variation

Table 2, shows the numerical analysis of the above results. To analyze the consistency and accuracy of PSO approach, the mean and standard deviation are also calculated for each variable. It is evident from the table that, the desired injection criteria, as per table 1, are achieved satisfactorily.

Table 2: Numerical Results

Variables	Δx_f	$\Delta x_{t_f}(best)$	$\Delta x_{t_f}(mean)$	$\Delta x_{t_f}(st.d)$	Units
t	5	147.6	148.8	0.94	s
r	20	8.684	16.745	2.7485	m
v	10	1.684	8.8539	4.8943	m/s
γ	-	0	0	0	deg

CONCLUSIONS

The application of Particle Swarm Optimization (PSO), in solving minimum time trajectory optimization problem is studied in this paper. The gravity turn trajectory is formulated for giving initial and final conditions. The target is achieved by minimum time and minimum error, at the injection point. Appropriate constraints are followed throughout the trajectory and necessary boundary conditions are applied. The PSO approach is implemented, for solving the complex nonlinear equations and the corresponding results, obtained through 50 iterations were validated. From the results, it is confirmed that, the injection parameters were met satisfactorily, as the future work hybrid approaches can be implemented and performance can be analysed.

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